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ABSTRACT

Seventy-two Ss attempted to solve four "who-done-it" type deductive reasoning problems which varied in mode of representation, type of logical connective employed, and affirmative versus negative statement of information. Affirmative and conjunctive problems were solved more frequently than negative and disjunctive problems (all are defined in the text of the articles); however, the mode of representation in which the problem was presented had no effect. Analysis of the mode of representation actually employed (based on Ss' written protocols) revealed a marked tendency for Ss in various conditions to change the original representation of the problem. When these changes were taken cognisance of, the matrix representation was superior to other forms on all but negatively stated problems. The somewhat poorer performance of females on this task was related to their greater reluctance at changing the representation of the problems. The original hypothesis of the instrumental role of representation of information in problem-solving was supported. Suggestions are offered for subsequent investigation. (Author/KS)

REPRESENTATION IN DEDUCTIVE PROBLEM-SOLVING: THE MATRIX[1]

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In a previous study, Schwartz (1971), presented a number of "who-done-it" deductive reasoning problems in sentance form to group of 30 adult Ss. An analysis of the S's protocols revealed a number of "modes of representation" by which Ss organized the information presented in the problems. One form - the matrix - was clearly superior to other representations for achieving solutions to affirmatively stated problems. Success in these tasks appeared more closely linked to the ability to accurately represent information in some suitable format rather than ability to draw logical conclusions from simple combinations of propositions. Task variables, (such as number of relevant dimensions, logical connective employed, proportion affirmative instances) known to have considerable effect on the solution rates for traditional concept-attainment tasks (Bourne, Ekstrand, & Dominowski, 1971) were effective to the extent to which they influenced the mode of representation Ss employed. Only where differential use of the matrix representation occured (in affirmative versus negative problems) were different solution rates observed.

The present study attempted to replicate these findings within a more structured task. We experimentally manipulated the mode of representation of information presented to the subject in order to determine more precisely the relationship between representation and problem-solving performance. It was hypothesized that Ss presented problems in the matrix mode of representation would exhibit superior performance.

Insert Figure 1 about here

Method. - Seventy-two Ss (36 males and females) were randomly drawn from the subject pool at a large urban university and tested for about two hours in groups of 12-15. All Ss attempted to solve four "who-done-it" type deductive problems (Fig.1) each containing information about five values on three dimensions (e.g., patients' name, room number, and illness). Two problems contained information presented in an affirmative fashion (e.g., the redhead bought the mini-skirt), the other two contained almost exclusively negative information (e.g., the maid was not in the bedroom). One problem in each of the above sets utilized conjunctive connectives (e.g., Ed had coffee and pie); the other, disjunctive connectives (Dave either wears a red or a blue tie). All problems contained sufficient information to determine all contingencies.

The design was a mixed factorial one with all Ss attempting to solve an affirmative and negative, conjunctive and disjunctive problem (four problems in all).

Mode of representation served as a between Ss factor with one-third of the S's receiving problems in a matrix format (Fig. 1a), one-third in a sentence format (Fig. 1b), and one-third in a graphic format (Fig. 1c).

(These modes had been observed in the previous study already mentioned).

In order to assure that Ss understood the problem information in the mode presented to him each S had to successfully complete a brief programed instruction sequence. This required he indicate in terms of equivalent english sentences the meaning of propositions presented in some mode of representation (Fig. 1). Ss were encouraged to show all work and had a maximum of 20 minutes to attempt to solve each of the four problems. If he solved in less than the maximum allowed time he went on to the next problem. The order of problems was randomized. After four problems, each S took a brief multiple choice logic test which was designed to indicate the extent to which he could draw valid implications from single affirmative or negative, conjunctive and disjunctive statements

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Insert Table 1 about here

Results. - 1. Effects of independent variable on solution rate - The percentage of Ss attaining solution under each condition is presented in Table 1. There are significant effects due to type of logical connective F (1,66) = 42.48, $p < .001$, and whether the information was presented in affirmative or negative form F (1,66) = 34.96 $p < .001$, however, contrary to expectations, mode of representation had virtually no effect on solution rate ($F < 1$). There was a tendency for males to do better than females F (1,66) = 4.85, $p < .05$, as well as a significant interaction between logical type of problem and affirmative-negative information F (1,66) = 5,44 $p < .05$. Although the affirmative problems were easier under all conditions, the advantage was greater on conjunctive as compared to disjunctive problems. No other effects were significant.

2. Analysis of representations actually used - In view of the unexpected findings of no effects for mode of representation we checked each of the protocols and classified them according to mode of representation actually used by each S in attempting to solve the problems. A classification scheme similar to that reported by Schwartz (1971) was employed. Tables 2 and 3 summarize the results of this analysis.

Insert Tables 2 and 3 about here

Table 2 indicates rather dramatically that Ss tend to be active participants when attempting to solve these problems with almost half the problems changed in representation from the mode in which they were originally presented. Note the highly significant effect due to mode of representation $F(2,66) = 22.12$, $p < .001$, with only 17% of the Ss in the matrix condition changing representation compared to 57% and 74% changes under the sentence and network treatments. Males tend to change representations somewhat more often than females 53% ro 46%, $F(1,66) = 7.43$, $p < .01$. All other effects were not significant.

A more detailed analysis of these changes of representation and subsequent solution rates is presented in Table 3. When mode of representation actually used is considered there is a highly significant effect $\chi^2(1) = 8.26$, $p < .01$ with the matrix representation yielding better solution rates than the other modes of representation.

In comparing the use and solution rates for the matrix representation versus the three other modes combined, we find the matrix was used about equally often in affirmative and negative problems (56 and 63 percent respectively). However, difference in soltuin rates between the matrix and other modes in the affirmative condition was highly significant. $Z = 4.55$, $p < .001$.

No such difference was found on negative problems ($\underline{Z} < 1$). Thus the advantage of the matrix representation appeared almost entirely in the affirmative problems.

There was no difference in use of matrix representation on conjunctive versus disjunctive problems with the matrix representation yielding superior performance on both types of problems Z's = 2.10 & 2.41 $\underline{p} < .05$. The matrix representation appeared equally advantageous in both types of problems compared to the other representations ($\underline{Z} < 1$).

As expected, the matrix was used more frequently in the condition where the problems were presented originally in that format (83%) compared to conditions where problems were presented in sentence (59%) or network formats (68%) $X^2(2) = 46.1$, $\underline{p} < .001$. Although the matrix representation produces higher solution rates under each condition the relative superiority of the matrix over the other representations remains about the same (Z's < 1) for Matrix-Sentence, Sentence-Network, and Matrix-Network comparisons.

Males appear to use the matrix representation more often than females $X^2(1) = 5.66$, $\underline{p} < .05$. Although the matrix representation results in significantly higher success rates than other representations for males ($\underline{Z} = 2.59$, $\underline{p} < .01$), and only approaches signficance in the female population ($\underline{Z} = 1.46$, $p < .10$), there is no significant difference in the relative advantage in solution rates of matrix versus other modes of representation between males and females ($\underline{Z} < 1$).

To summarize, Table 3 indicates superior performance for those problems attempted within the framework of a matrix representation in all conditions except for negatively stated problems.

Insert Table 4 about here

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3. Other transformations - In addition to changes in mode of representation, two other transformations were determined from the S's protocals. Table 4 indicates the number of problems where Ss changed either the logical connective or the affirmative-negative aspects of the problem. In 45 problems, negative information was transformed into affirmative statements, compared to only 2 such cases in the opposite direction. Likewise, in 54 disjunctive problems information was changed into a conjunctive form, compared to only 17 conjunctive to disjunctive transformations. Finally, there is a trend to more readily change negative problems than affirmatively stated problems $X^2(1) = 2.8$ $p < .10$, from conjunctive to disjunctive form. Performance in the logic post-test accounts for only 10% of the variance in success on the four experimental problems $r(142) = 34$, $p < .01$. This is of the same magnitude as the .28 correlation between success on "who-done-it" problems and performance on this logic test found previously (Schwartz, 1971).

Discussion - The main findings of the previous study (Schwartz, 1971) are confirmed in that the matrix mode of representation as actually used by Ss, again leads to significantly better performance on all problems except those where information was presented in terms of negative statements. Furthermore S's seem to recognize the appropriatemenss of this form of representation as evidenced in the low rate of changes in representation (17%) in the matrix as compared to the sentence and network conditions (57% and 74%). When Ss do change the representation of the information they most often change to a matrix representation; thus of the problems originally presented in sentence or network modes almost half (46%) were changed by S's into a matrix representation.

The generally superior performance on affirmative and conjunctive problems is in agreement with previous findings in the concept attainment

literature (Conant& Trabasso, 1964; Schvaneldt, 1966). That the advantage of affirmative over negative problems is greater in the conjunctive compared to disjunctive case also corresponds to results found in studies on concept attainment (Bourne & Guy, 1968). Positive instances are more readily utilized in most strategies for attaining conjunctive concepts while the reverse is true for disjunctive concepts. As has been noted previously (Schwartz, 1971) these results must be interpreted with caution since the problems were not precisely equated in terms of sentence by sentence information correspondence. Instead, since the primary objective of the study was to exlore the utility of various modes of representation, the problems were matched only to the extent that each contained sufficient information for solution, approximately an equal number of statements, and specified five values on each of four dimensions.

One reason for the poorer performance on negative problems can be infered from the data in Table 4. Forty-five negative problems were transformed by Ss into an affirmative mode while the reverse occured only twice. This extra operation of transferring information provides additional opportunities for errors to take place thus resulting in a lower overall success rate. An intriguing ancillary finding in this study appears with respect to the small, but significantly superior performance of males (56%) compared to females (45%). Previous studies on problem-solving have reported similar results (Duncan, 1961; Stoats, 1957). If, as hypothesized, development of an appropriate representation of information plays a key role in many problem-solving situations, then the tendency for females to be somewhat more reluctant than males (46% - 53%) to change the problem from its original representation into a new mode may contribute to the small but consistent difference between performance of the sexes.

To conclude, the hypotheses of the instrumental role of representation of information in problem-solving was supported. The superiority of a matrix mode of representation for a variety of problems has been demonstrated in two studies suggesting the need for subsequent investigation concerning why this form of representation is effective, and exploring tasks for which other representatives may be more suitable.

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Footnotes

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Table 1

Percentage of Ss Attaining Solution Under Each Condition (n=24 Per Cell)

Mode of Representation	Problem Type: Conjunctive Affirmative	Conjunctive Negative	Disjunctive Affirmative	Disjunctive Negative	All Prob. Male	All Prob. Female	Totals
Sentence	79%	46%	46%	25%	60%	42%	51%
Matrix	83%	50%	46%	25%	54%	46%	50%
Net	88%	33%	42%	38%	54%	48%	51%
Totals	83%	43%	45%	29%	56%	45%	51%

Table 2

Percentage of Ss Changing Representation Under Each Condition (n=24 per Cell)

Mode of Representation Presented	Problem Type: Conjunctive		Disjunctive		All Prob. Male	All Prob. Female	Totals All Probs.
	Affirmative	Negative	Affirmative	Negative			
Sentence	42%	58%	58%	71%	63%	52%	57%
Matrix	8%	8%	21%	29%	17%	17%	17%
Net	79%	71%	58%	79%	79%	68%	74%
Totals	43%	46%	46%	60%	53%	46%	49%

Table 3

Proportion of Times Each Mode of Representation was Used and Corresponding Solution Rates for Various Problems and Populations

Mode of Representation	Type of Problem: AFF. (144 Prob)		NEG. (144 Prob)		Conj. (144 Prob)		Disj. (144 Prob)		Matrix (96 Prob)		Sentence (96 Prob)		Network (96 Prob)		Male (144 Prob)		Female (144 Prob)		TOTAL (288 Prob)	
	use	sol.	use	sol.	use	sol.	use	sol.	use	sol.	use	sol.	use	sol.	use	sol.	use	sol.	use	sol.
1. MATRIX	.56	.80	.63	.37	.59	.71	.60	.45	.83	.53	.56	.59	.35	.68	.69	.64	.50	.51	.59	.58
2. Sentence	.26	.49	.22	.36	.30	.60	.18	.20	.17	.38	.39	.41	.19	.58	.24	.46	.24	.44	.24	.45
3. Network	.15	.43	.13	.22	.10	.42	.18	.28	.00	--	.00	--	.42	.31	.06	.33	.22	.30	.14	.31
4. Other	.03	.33	.02	.50	.01	.00	.03	.33	.00	--	.04	.25	.03	.00	.03	.50	.02	.33	.03	.35

Table 4

Number of Problems showing Evidence of Conjunctive - Disjunctive or Positive - Negative Transformation

Original Form of Problem	Form Changed To: Conjunctive	Disjunctive	Affirmative	Negative
Conjunctive		17	25	2
Disjunctive	54		20	0
Affirmative	32	6		2
Negative	22	11	45	

Figure Captions

Figure 1. Sample Problems

Figure 1(a) Sentence Representation

Figure 1(b) Matrix Representation

Figure 1(c) Network Representation

1(a)

Five men are in a hospital. Each one is suffereing from a different disease.

1. Mr. Wilson is in Room 102.
2. Mr. Alex has cancer.
3. Mr. Thomas is in Room 101.
4. One of the men has epilepsy.
5. The man with mononucleosis is in Room 104.
6. Mr. Young is one of the patients.
7. The man with asthma is in Room 101.
8. Mr. Wilson has T.B.
9. Mr. Osborn is in Room 105.
10. One of the patients is in Room 103.

What disease does Mr. Young have?

Figure 1

1(b)

Five men are in a hospital. Each one is suffering from a different disease.

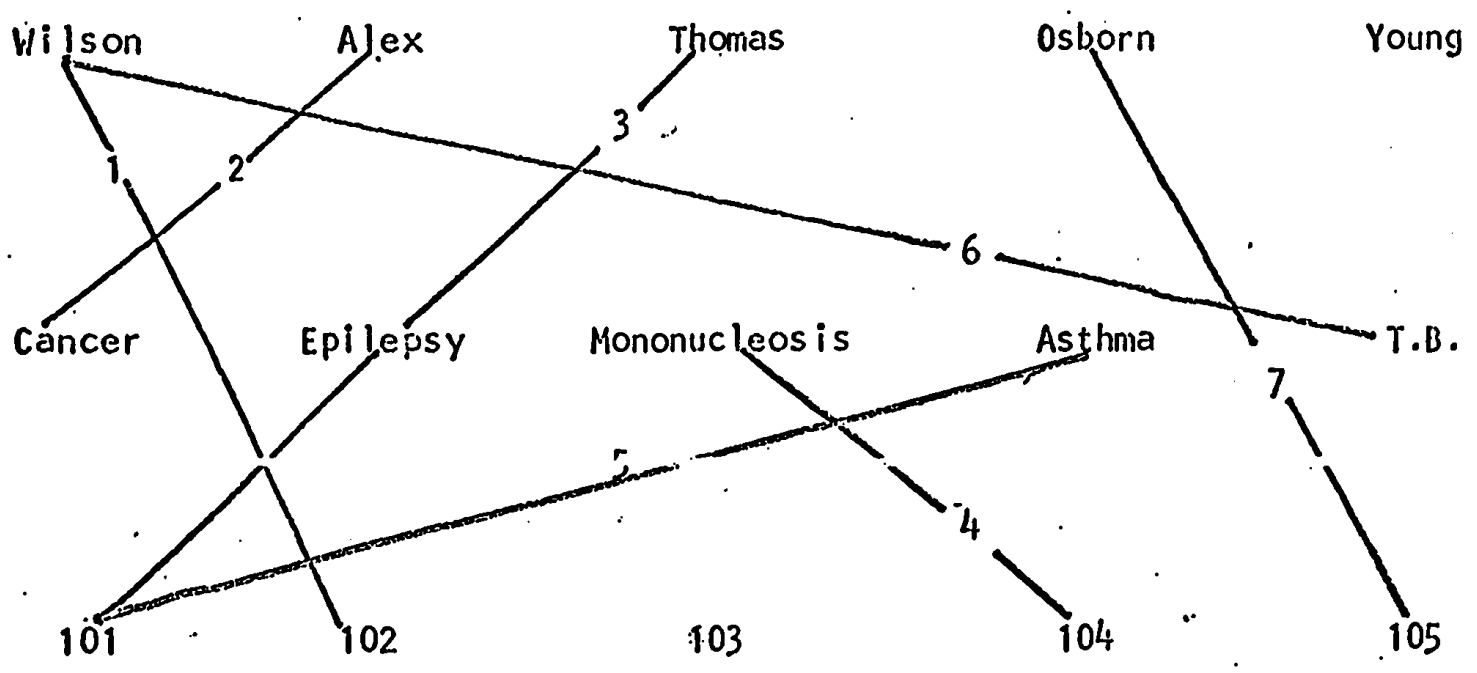
Name	Disease	Room
1. Mr. Wilson		102
2. Mr. Alex	Cancer	
3. Mr. Thomas		101
4.	Epilepsy	
5.	Mononucleosis	104
6. Mr. Young		
7.	Asthma	101
8. Mr. Wilson	T.B.	
9. Mr. Osborn		105
10.		103

What disease does Mr. Young have?

Figure 1

1(c)

Five men are in a hospital. Each one is suffering from a different disease.



What disease does Mr. Young have?

Figure 1